**CBSE Class 10 Mathematics**

**Important Questions**

**Chapter 1**

**Real Numbers**

**1 Marks Questions**

**1. Use Euclid's division lemma to show that the square of any positive integer is either of form 3*m* or 3*m* + 1 for some integer *m*.**

**[Hint: Let *x* be any positive integer then it is of the form 3*q*, 3*q* + 1 or 3*q* + 2. Now square each of these and show that they can be rewritten in the form 3*m* or 3*m* + 1.]**

**Ans.** Let $a$ be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 \ or (3q+1)^2 \ or (3q+2)^2$$
$$a^2 = (9q)^2 \ or 9q^2 + 6q + 1 \ or \ 9q^2 + 12q + 4$$
$$= 3 \times (3q^2) \ or \ 3(3q^2 + 2q) + 1 \ or \ 3(3q^2 + 4q + 1) + 1$$
$$= 3k_1 \ or \ 3k_2 + 1 \ or \ 3k_3 + 1$$

Where $k_1$, $k_2$, and $k_3$ are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3*m* or 3*m* + 1.

**2. Express each number as product of its prime factors:**

**(i) 140**

**(ii) 156**

**(iii) 3825**

**(iv) 5005**

**(v) 7429**

**Ans. (i)** 140 = 2 ´ 2 ´ 5 ´ 7 = $2^{2}$´ 5 ´ 7

**(ii)** 156 = 2 ´ 2 ´ 3 ´ 13 = $2^{2}$´ 3 ´ 13

**(iii)** 3825 = 3 ´ 3 ´ 5 ´ 5 ´ 17 = $3^{2}$´ $5^{2}$´ 17

**(iv)** 5005 = 5 ´ 7 ´ 11 ´ 13

**(v)** 7429 = 17 ´ 19 ´ 23

**3. Given that HCF (306, 657) = 9, find LCM (306, 657).**

**Ans.** HCF (306, 657) = 9

We know that, LCM × HCF = Product of two numbers

\ LCM × HCF =306 × 657

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

LCM = 22338

**4. Check whether $6^n$ can end with the digit 0 for any natural number *n*.**

**Ans.** If any number ends with the digit 0, it should be divisible by 10.

In other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of $6^n$.

Hence, for any value of $n$, $6^n$ will not be divisible by 5.

Therefore, $6^n$ cannot end with the digit 0 for any natural number $n$.

**5. Prove that (3+2 $\sqrt{5}$ ) is irrational.**

**Ans.** We will prove this by contradiction.

Let us suppose that (3+2 $\sqrt{5}$ ) is rational.

It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$\frac{a}{b} = 3 + 2\sqrt{5}$$

$$\Rightarrow \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \quad \textbf{... (1)}$$

***a* and *b*** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible.
Therefore, our supposition is wrong. (3+2 $\sqrt{5}$ ) cannot be rational.

Hence, (3+2 $\sqrt{5}$ ) is irrational.

**6.** $7 \times 11 \times 13 \times 15 + 15$ **is a**

**(a) Composite number**

**(b) Whole number**

**(c) Prime number**

**(d) None of these**

**Ans.** (a)and(b)both

**7. For what least value of 'n' a natural number, $(24)^n$ is divisible by 8?**

**(a) 0**

**(b) -1**

**(c) 1**

**(d) No value of 'n' is possible**

**Ans. (c)** 1

**8. The sum of a rational and an irrational number is**

**(a) Rational**

**(b) Irrational**

**(c) Both (a) & (c)**

**(d) Either (a) or (b)**

**Ans. (b)** Irrational

**9. HCF of two numbers is 113, their LCM is 56952. If one number is 904, then other number is:**

**(a)** 7719

**(b)** 7119

**(c)** 7791

**(d) 7911**

**Ans. (b)** 7119

**10. A lemma is an axiom used for proving**

**(a) other statement**

**(b) no statement**

**(c) contradictory statement**

**(d) none of these**

**Ans. a)** other statement

**11. If HCF of two numbers is 1, the two numbers are called relatively _______ or _______.**

**(a) prime, co-prime**

**(b) composite, prime**

**(c) Both (a) and (b)**

**(d) None of these**

**Ans. (a)** prime, co-prime

**12.** $2.\overline{35}$ **is**

**(a) a terminating decimal number**

**(b) a rational number**

**(c) an irrational number**

**(d) Both (a) and (b)**

**Ans. (b)** a rational number

**13. 2.13113111311113......is**

**(a) a rational number**

**(b) a non-terminating decimal number**

**(c) an irrational number**

**(d) Both (a) & (c)**

**Ans. (c)** an irrational number

**14. The smallest composite number is:**

**(a) 1**

**(b) 2**

**(c) 3**

**(d) 4**

**Ans. (c)** 3

**15.** $1.23\overline{48}$ **is**

**(a) an integer**

**(b) an irrational number**

**(c) a rational number**

**(d) None of these**

**Ans. (c)** a rational number

**16.** $\pi$ **is**

**(a) a rational number**

**(b) an irrational number**

**(c) both (a) & (b)**

**(d) neither rational nor irrational**

**Ans. (b)** an irrational number

**17. (2+ $\sqrt{5}$ ) is**

**(a) a rational number**

**(b) an irrational number**

**(c) an integer**

**(d) not real number**

**Ans. (b)** an irrational number

# CBSE Class 10 Mathematics
# Important Questions
# Chapter 1
# Real Numbers

## 2 Marks Questions

**1. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.**

**Ans .** Let $a$ be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where $k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2\,(3q + 1) + 1 = 2k_2 + 1$, where $k_2$ is an integer

$6q + 5 = (6q + 4) + 1 = 2\,(3q + 2) + 1 = 2k_3 + 1$, where $k_3$ is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where $k$ is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,

Or $6q + 5$

**2. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**

**Ans.** We have to find the HCF(616, 32) to find the maximum number of columns in which they can march.

To find the HCF, we can use Euclid's algorithm.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

**3. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9*m*, 9*m* + 1 or 9*m* + 8.**

**Ans.** Let $a$ be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

a = 3q or 3q + 1 or 3q + 2

Therefore, every number can be represented as these three forms.

We have three cases.

**Case 1**: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where $m$ is an integer such that $m = 3q^3$

**Case 2**: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where $m$ is an integer such that $m = (3q^3 + 3q^2 + q)$

**Case 3**: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where $m$ is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

**4. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.**

**(i) 26 and 91**

**(ii) 510 and 92**

**(iii) 336 and 54**

**Ans. (i)** 26 and 91

26= 2 × 13

91 = 7 × 13

HCF = 13

LCM = 2 × 7 × 13 = 182

Product of two numbers 26 and 91 = 26 × 91 = 2366

HCF × LCM = 13 × 182 = 2366

Hence, product of two numbers = HCF × LCM

**(ii)** 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF = 2

LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of two numbers 510 and 92 = $510 \times 92 = 46920$

HCF × LCM = $2 \times 23460 = 46920$

Hence, product of two numbers = HCF × LCM

**(iii)** 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

HCF = $2 \times 3 = 6$

LCM = $2^4 \times 3^3 \times 7 = 3024$

Product of two numbers 336 and 54 = $336 \times 54 = 18144$

HCF × LCM = $6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

**5. Find the LCM and HCF of the following integers by applying the prime factorisation method.**

**(i) 12, 15 and 21**

**(ii) 17, 23 and 29**

**(iii) 8, 9 and 25**

**Ans. (i)** 12, 15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF = 3

LCM = $2^2 \times 3 \times 5 \times 7 = 420$

**(ii)** 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

HCF = 1

LCM = $17 \times 23 \times 29 = 11339$

**(iii)** 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

HCF = 1

LCM = $2^3 \times 3^2 \times 5^2 = 1800$

**6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.**

**Ans.** Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors.

Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

Hence, it is a composite number.

**7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**

**Ans.** It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 × 3 × 3 And, 12 = 2 × 2 × 3

LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36

Therefore, Ravi and Sonia will meet together at the starting pointafter 36 minutes.

**8. Prove that $\sqrt{5}$ is irrational.**

**Ans.** Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers ***a* and *b*** ($b \neq 0$)

such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ ... **(1)**

It means that 5 is factor of $a^2$

Hence, **5 is also factor of *a*** by Theorem. ... **(2)**

If, **5 is factor of *a***, it means that we can write ***a*=5*c*** for some integer ***c***.

Substituting value of ***a*** in **(1)**,

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

It means that 5 is factor of $b^2$.

Hence, **5 is also factor of *b*** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that **5** is factor of both ***a* and *b***.

But, ***a* and *b*** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

**9. Write down the decimal expansions of those rational numbers in Question 1 which have terminating decimal expansions.**

**Ans. (i)** $\frac{13}{3125}=\frac{13}{5^5}=\frac{13\times 2^5}{5^5\times 2^5}=\frac{13\times 2^5}{10^5}\frac{416}{10^5}=0.00416$

**(ii)** $\frac{17}{8}=\frac{17}{2^3}=\frac{17\times 5^3}{2^3\times 5^3}=\frac{17\times 5^3}{10^3}=\frac{2125}{10^3}=2.215$

**(iii)** $\frac{15}{1600}=\frac{15}{2^6\times 5^2}=\frac{15\times 5^4}{2^6\times 5^2\times 5^4}=\frac{15\times 5^4}{10^6}=\frac{9375}{10^6}=0.009375$

**(iv)** $\frac{23}{2^3\times 5^2}=\frac{23\times 5^1}{2^3\times 5^2\times 5^1}=\frac{23\times 5^1}{10^3}=\frac{115}{10^3}=0.115$

**(v)** $\frac{6}{15}=\frac{2}{5}=\frac{2\times 2}{5\times 2}=\frac{4}{10}=0.4$

**(vi)** $\frac{35}{50}=\frac{7}{10}=0.7$

**10. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If, they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?**

**(i) 43.123456789**

**(ii) 0.120120012000120000...**

**(iii)** $43.\overline{123456789}$

**A ns. (i)** 43.123456789

It is rational because decimal expansion is terminating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of $q$ are of the form $2^n \times 5^m$ where $n$ and $m$ are non-negative integers.

**(ii)** 0.120112001200012000...

It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

**(iii)** $43.\overline{123456789}$

It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of $q$ **are not** of the form $2^n \times 5^m$ where $n$ and $m$ are non-negative integers.

**11. Show that every positive even integer is of the from 2q and that every positive odd integer is of the form 2q + 1 for some integer q.**

**Ans.** Let a = bq + r : b = 2

$0 \leq r < 2$ i.e., r = 0, 1

a = 2q + 0, 2q + 1,

If a = 2q (which is even)

If a = 2q + 1 (which is odd)

So, every positive even integer is of the form 2q and odd integer is of the form 2q + 1.

**12. Show that any number of the form $4^n$, n e N can never end with the digit 0.**

**Ans.** $4^n = \left[2^2\right]^n = 2^{2n}$

It does not contains '5'. So $4^n, n \in N$ *can never end with the digit* 0.

**13. Use Euclid's Division Algorithm to find the HCF of 4052 and 12576.**

**Ans.** $12576 = 4052 \times 3 + 420$

$$4052 = 420 \times 9 + 272$$
$$420 = 272 \times 1 + 148$$
$$272 = 148 \times 1 + 124$$
$$148 = 124 \times 1 + 24$$
$$124 = 24 \times 5 + 4$$
$$24 = 4 \times 6 + 0$$

HCF of 12576 and 4052 is '4'.

**14. Given that HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.**

**Ans.**

$$HCF \times LCM = a \times b$$
$$\Rightarrow 23 \times 1449 = 161 \times b$$
$$\Rightarrow b = \frac{23 \times 1449}{161} = 207$$
$$\therefore \textit{Other number is } 207$$

**15. Show that every positive odd integer is of the form (4q + 1) or (4q + 3) for some integer q.**

**Ans.** Let a = 4q + r : $0 \leq r < 4$

$\therefore a = 4q = 2(2q)$ an even integer

$a = 4q + 1 = 2(2q) + 1$ an odd integer

$a = 4q + 2 = 2(2q + 1)$ an even integer

$a = 4q + 3 = 2(2q + 1) + 1$ an odd integer

$\therefore$ Every positive odd integer is of the form

$(4q + 1) or (4q + 3)$ for some integer

**16. Show that any number of the form $6^x$, $x \in N$ can never end with the digit 0.**

**Ans.** $6^n = (2 \times 3)^n = 2^n \times 3^n$

$\therefore$ *5 is not a factor of* $6^n$

$\therefore$ *It never ends with* 0.

**17. Find HCF and LCM of 18 and 24 by the prime factorization method.**

**Ans.** $18 = 2 \times 3 \times 3 = 2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$\therefore HCF = 2 \times 3 = 6$

**18. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.**

**Ans.** $LCM \times HCF = a \times b$

$\Rightarrow 1449 \times 23 = 161 \times b$

$$\Rightarrow b = \frac{1449 \times 23}{161} = 207$$

**19. Prove that the square of any positive integer of the form 5g + 1 is of the same form.**

**Ans.** $a = 5q + 1$

$\therefore\ a^2 = (5q+1)^2$

$= 25q^2 + 10q + 1$

$= 5(5q^2 + 2q) + 1$

$= 5m + 1$

**20. Use Euclid's Division Algorithm to find the HCF of 4052 and 12576.**

**Ans.** *HCF of* 12576 *and* 4052

$12576 = 4052 \times 3 + 420$

$4052 = 420 \times 9 + 272$

$420 = 272 \times 1 + 148$

$272 = 148 \times 1 + 124$

$148 = 124 \times 1 + 24$

$124 = 24 \times 5 + 4$

$24 = 4 \times 6 + 0$

$\therefore\ HCF = 4$

**21. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.**

**Ans.** The required number is the HCF of (245 – 5) and (1029 – 5) i.e., 240 and 1024.

$1024 = 240 \times 4 + 64$

$240 = 64 \times 3 + 48$

$64 = 48 \times 1 + 16$

$48 = 16 \times 3 + 0$

$\therefore HCF$ *is* 16.

**22. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin?**

**Ans.** The required greatest capacity is the HCF of 120, 180 and 240

$240 = 180 \times 1 + 60$

$180 = 60 \times 3 + 0$

$HCF\ is\ 60.$

$Now\ HCF\ of\ 60,\ 120$

$120 = 60 \times 2 + 0$

$\therefore HCF\ of\ 120,\ 180\ and\ 240\ is\ 60.$

$\therefore$ The required capacity *is* 60 litres.

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**Chapter 1**
**Real Numbers**

**3 Marks Questions**

**1. Use Euclid's division algorithm to find the HCF of:**

**(i) 135 and 225**

**(ii) 196 and 38220**

**(iii) 867 and 255**

**Ans. (i)** 135 and 225

We have 225 > 135,

So, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Here remainder $90 \neq 0$, we apply the division lemma again to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder $45 \neq 0$, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since that time the remainder is zero, the process get stops.

The divisor at this stage is 45

Therefore, the HCF of 135 and 225 is 45.

**(ii)** 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

**(iii)** 867 and 255

We have 867 > 255,

So, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Here remainder $102 \neq 0$, we apply the division lemma again to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

Here remainder $51 \neq 0$, we apply the division lemma again to 102 and 51 to obtain

$102 = 51 \times 2 + 0$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

**2. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.**

**Ans.** The greater number of 6 digits is 999999.

LCM of 24, 15, and 36 is 360.

$$999999 = 360 \times 2777 + 279$$

$$\text{Required number is} = 999999 - 279 = 999720$$

**3. Prove that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.**

**Ans.** Let a = 4q + r, when r = 0, 1, 2 and 3

$\therefore$ Numbers are 4q, 4q + 1, 4q + 2 and 4q + 3

$$x^2 + y^2$$

**4. 144 cartons of coke can and 90 cartons of Pepsi can are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of the same drink. What would be the greater number of cartons each stack would have?**

**Ans.** We find the HCF of 144 and 90

$$144 = 90 \times 1 + 54$$
$$90 = 54 \times 1 + 36$$
$$54 = 36 \times 1 + 18$$
$$36 = 18 \times 2 + 0$$
$$\therefore \quad HCF = 18$$

$$\text{So, greatest number of cartons is } 18.$$

**5. Prove that product of three consecutive positive integers is divisible by 6.**

**Ans.** Let three consecutive numbers be x, (x + 1) and (x + 2)

Let x = 6q + r; $0 \leq r < 6$

$$\therefore x = 6q,\ 6q+1,\ 6q+2,\ 6q+3,\ 6q+4,\ 6q+5$$

$product\ of\ x(x+1)(x+2) = 6q(6q+1)(6q+2)$

$if\ x = 6q\ then\ which\ is\ divisible\ by\ 6$

$if\ x = 6q+1$

$= (6q+1)(6q+2)(6q+3)$

$= 2(3q+1).3(2q+1)(6q+1)$

$= 6(3q+1).(2q+1)(6q+1)$

$which\ is\ divisible\ by\ 6$

$if\ x = 6q+2$

$= (6q+2)(6q+3)(6q+4)$

$= 3(2q+1).2(3q+1)(6q+4)$

$= 6(2q+1).(3q+1)(6q+1)$

$which\ is\ divisible\ by\ 6$

$if\ x = 6q+3$

$= (6q+3)(6q+4)(6q+5)$

$= 6(2q+1)(3q+2)(6q+5)$

$which\ is\ divisible\ by\ 6$

$if\ x = 6q+4$

$= (6q+4)(6q+5)(6q+6)$

$= 6(6q+4)(6q+5)(q+1)$

$which\ is\ divisible\ by\ 6$

$if\ x = 6q+5$

$= (6q+5)(6q+6)(6q+7)$

$= 6(6q+5)(q+1)(6q+7)$

$which\ is\ divisible\ by\ 6$

$\therefore\ product\ of\ any\ three\ natural\ numbers\ is\ divisible\ by\ 6.$

**6. Prove that (3 – $\sqrt{5}$ ) is an irrational number.**

**Ans.** $Let\ 3-\sqrt{5}=\frac{p}{q}$

$x^2+y^2$

$(3q-p)$ and $q$ are integers, so $\left(\frac{3q-p}{q}\right)$ is a rational number, but $\sqrt{5}$ is an irrational number. This contradiction arises because of our wrong assumption.

So $\left(3-\sqrt{5}\right)$ is an irrational number.

**7. Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.**

**Ans.** Let x = 2p + 1 and y = 2q + 1

$\therefore x^2+y^2=(2p+1)^2+(2q+1)^2$

$=4p^2+4p+1+4q^2+4q+1$

$=4\left(p^2+q^2+p+q\right)+2$

$=2\left(2p^2+2q^2+2p+2q+1\right)$

$=2m$ where $m=\left(2p^2+2q^2+2p+2q+1\right)$

$\therefore x^2+y^2$ is an even number but not divisible by 4.

**8. Show that one and only one out of n, (n + 2) or (n + 4) is divisible by 3, where n $\in$ N.**

**Ans.** Let the number be (3q + r)

If $n = 3q$ then, numbers are $3q$, $(3q+1)$, $(3q+2)$

$3q$ is divisible by 3.

If $n = 3q+1$ then, numbers are $(3q+1)$, $(3q+3)$, $(3q+4)$

$(3q+3)$ is divisible by 3.

If $n = 3q+2$ then, numbers are $(3q+2)$, $(3q+4)$, $(3q+6)$

$(3q+6)$ is divisible by 3.

$\therefore$ out of $n$, $(n+2)$ and $(n+4)$ only one is divisible by 3.

**9. Use Euclid's Division Lemma to show that the square of any positive integer of the form 3m or (3m + 1) for some integer q.**

**Ans.** $Let\ a = 3q + r \quad ; \quad 0 \le r < 3$

$\therefore\ a = 3q,\ 3q+1\ and\ 3q+2$

$a^2 = (3q)^2 = 9q^2 = 3(3q)^2 = 3m\ \left(where\ m = 3q^2\right)$

$= 3m\left(where\ m = 3q^2 + 4q + 1\right)$

**10. Prove that in $\sqrt{n}$ is not a rational number, if n is not perfect square.**

**Ans.** Let $\sqrt{n}$ be a rational number.

$\therefore \sqrt{n} = \frac{p}{q}$

$\Rightarrow n = \frac{p^2}{q^2}$

$p^2 = nq^2$

$\Rightarrow n$ divides $p^2$

$\Rightarrow n$ divides $p \rightarrow (i)$

Let $p = nm$

$\Rightarrow p^2 = n^2m^2$

$\therefore n^2m^2 = nq^2$

$q^2 = nm^2$

$\Rightarrow n$ divides $q^2$

$\Rightarrow n$ divides $q \rightarrow (ii)$

from (i) and (ii) n is a common factor of both $p$ and $q$.

this contradicts the asssumption that $p$ and $q$ are co-prime.

So, our supposition is wrong.

**11. Prove that the difference and quotient of $(3+2\sqrt{3})$ and $(3-2\sqrt{3})$ are irrational.**

**Ans.** Difference of $(3+2\sqrt{3})$ and $(3-2\sqrt{3})$

$= (3+2\sqrt{3}) - (3-2\sqrt{3})$

$= 3+2\sqrt{3}-3+2\sqrt{3}$

$= 4\sqrt{3}$ which is irrational.

and quotient is

$= \frac{3+2\sqrt{3}}{3-2\sqrt{3}} \times \frac{3+2\sqrt{3}}{3+2\sqrt{3}}$

**12. Show that ($n^2$ – 1) is divisible by 8, if n is an odd positive integer.**

**Ans.** Let n = 4q + 1(an odd integer)

$\therefore\ n^2 - 1 = (4q+1)^2 - 1$

$= 16q^2 + 1 + 8q - 1$

$= 16q^2 + 8q$

$= 8(2q^2 + 2)$

$= 8m$, which is divisible by 8.

**13. Use Euclid Division Lemma to show that cube of any positive integer is either of the form 9m, (9m + 1) or (9m + 8).**

**Ans.** Let a = 3q + r; $0 \le r < 3$

$\therefore\ a = 3q$ ; then $a^3 = 27q^3 = 9m$ ; where $m = 3q^3$

when $a = 3q+1$ ; then $a^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 8$ (where m $= 3q^3 + 3q^2 + q$)

when $a = 3q+2$ ; then $a^3 = (3q+2)^2$

$= 27q^3 + 54q^2 + 36q + 8$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$= 9m + 8$ (where m $= 3q^3 + 6q^2 + 4q$)

Hence, cubes of any positive integer is either of the form $9m$, $(9m+1)$ or $(9m+8)$.

## CBSE Class 10 Mathematics
## Important Questions
## Chapter 1
## Real Numbers

### 4 Marks Questions

**1. Prove that the following are irrationals.**

**(i)** $\frac{1}{\sqrt{2}}$

**(ii)** $7\sqrt{5}$

**(iii)** $6+\sqrt{2}$

**Ans . (i)** We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers *a* and *b* (*b*≠0) such that

$\frac{1}{\sqrt{2}}$ =*ab*

$\Rightarrow \sqrt{2}=\frac{b}{a}$ ... **(1)**

**R.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

**(ii)** We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers *a* and *b* (*b*≠0) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b} \quad \textbf{... (1)}$$

**R.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

**(iii)** We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that ($6+\sqrt{2}$) is rational.

It means that we have co-prime integers ***a* and *b*** (*b*≠0) such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a-6b}{b} \quad \textbf{... (1)}$$

***a* and *b*** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6 + \sqrt{2}$) cannot be rational.

Hence, ($6 + \sqrt{2}$) is irrational.

**2. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating decimal expansion.**

**(i)** $\frac{13}{3125}$

**(ii)** $\frac{17}{8}$

**(iii)** $\frac{64}{455}$

**(iv)** $\frac{15}{1600}$

**(v)** $\frac{29}{343}$

**(vi)** $\frac{23}{2^3 \times 5^2}$

**(vii)** $\frac{129}{2^2 \times 5^7 \times 7^5}$

**(viii)** $\frac{6}{15}$

**(ix)** $\frac{35}{50}$

**(x)** $\frac{77}{210}$

**Ans.** According to Theorem, any given rational number of the form $\frac{p}{q}$ **where *p* and *q* are co-prime**, has a terminating decimal expansion if q is of the form $2^n \times 5^m$, where m and n are non-negative integers.

**(i)** $\frac{13}{3125}$

$q$=3125=5×5×5×5×5=$5^5$

Here, denominator is of the form $2^n \times 5^m$, where m=5 and n=0.

It means rational number $\frac{13}{3125}$ has a **terminating** decimal expansion.

**(ii)** $\frac{17}{8}$

$q$=8=2×2×2=$2^3$

Here, denominator is of the form $2^n \times 5^m$, where m=0 and n=3.

It means rational number $\frac{17}{8}$ has a **terminating** decimal expansion.

**(iii)** $\frac{64}{455}$

$q$=455=5×91

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{64}{455}$ has a **non-terminating repeating** decimal expansion.

**(iv)** $\frac{15}{1600} = \frac{3}{320}$

$q$=320=2×2×2×2×2×2×5 = $2^6 \times 5$

Here, denominator is of the form $2^n \times 5^m$, where m=1 and n=6.

It means rational number $\frac{15}{1600}$ has a **terminating** decimal expansion.

**(v)** $\frac{29}{343}$

$q$=343=7×7×7

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{29}{343}$ has **non-terminating repeating** decimal expansion.

**(vi)** $\frac{23}{2^3 \times 5^2}$

$q = 2^3 \times 5^2$

Here, denominator is of the form $2^n \times 5^m$, where m=2 and n=3 are non-negative integers.

It means rational number $\frac{23}{2^3 \times 5^2}$ has **terminating** decimal expansion.

**(vii)** $\frac{129}{2^2 \times 5^7 \times 7^5}$

$q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{129}{2^2 \times 5^7 \times 7^5}$ has **non-terminating repeating** decimal expansion.

**(viii)** $\frac{6}{15} = \frac{2}{5}$

$q$=5=$5^1$

Here, denominator is of the form $2^n \times 5^m$, where m=1 and n=0.

It means rational number $\frac{6}{15}$ has **terminating** decimal expansion.

**(ix)** $\frac{35}{50} = \frac{7}{10}$

$q$=10=2×5=$2^1 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m=1 and n= 1.

It means rational number $\frac{35}{50}$ has **terminating decimal** expansion.

**(x)** $\frac{77}{210} = \frac{11}{30}$

$q$=30=5×3×2

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{77}{210}$ has **non-terminating repeating** decimal expansion.